J. Myanmar Acad. Arts Sci. 2019 **Vol. XVII. No.2B**

CONSTRUCTING A COUPLED CHANNEL $\bar{K}$N-πΣ SEPARABLE POTENTIAL

Hnin Nu Aye[1], Thida Wint[2], Khin Swe Myint[3]

Abstract

The purpose of our research is to determine the $\bar{K}$N potential parameters which reproduce the experimental binding energy and level width of Λ(1405) resonance as well as the $\bar{K}$N scattering parameter. Λ(1405) has been interpreted as a $\bar{K}$N [1]quasi-bound state which is embedded in the πΣ continuum. Particle Data Group (PDG) interpreted on the experimental finding of K^-p bound state that Λ(1405) is located around 1405MeV. Chiral dynamics claimed that there appears two poles in $\bar{K}$N-πΣ coupled channel system. The first pole is located around 1420MeV mainly coupled to $\bar{K}$N channel and the second pole is located around 1320MeV mainly coupled to Σπ channel. The position of $\bar{K}$N bound state is a still controversial issue. In order to get information about the $\bar{K}$N interaction, the parameter set 1 which satisfies the PDG data and the parameter set 2 which satisfies the Chiral model's result for single channel are constructed by solving the Schrödinger equation with Yukawa-type separable potential. For the coupled-channel system, [2]the optical potential is constructed based on the Feshbath theory and the parameter set 1 and set 2 of the optical potential are investigated. In our research, we assumed that the range parameters Λ_1 and Λ_2 are the same and the interaction between πΣ-πΣ: $V_{22}=0$. Our calculated scattering length is consistent with the empirical value at the mass of exchanged boson; 600MeV/c^2 for set 1 and 670MeV/c^2 for set 2. For the coupled channel system, the parameter set 1 are $\Lambda = 3.041\text{fm}^{-1}$, $S_{11} = -1.074$, $S_{12} = 0.707$ and $U_0 = -3.063 – i0.466\text{MeV.fm}^3$ while the parameter set 2 are $\Lambda = 3.395\text{fm}^{-1}$, $S_{11} = -0.890$, $S_{12} = 0.733$ and $U_0 = -2.476 – i0.428\text{MeV.fm}^3$.

Keywords: quasi-bound state, the optical potential, $\bar{K}$N-πΣ separable potential, Λ(1405) resonance

Introduction

The study of the antikaon-nuclei has attracted a significant amount of attention in hadron and nuclear physics communities since over half a century ago. The properties of $\bar{K}$ nuclei can be attributed to the bare $\bar{K}$N interaction which is strongly attractive. According to a precise experiment of a kaonic

1. Demonstrator, Department of Physics, University of Magway
2. Lecturer, Department of Physics, University of Mandalay
3. Rector (Rtd.), Emeritus Professor, Department of Physics, University

hydrogen, it is suggested that the system of K^- and proton has a nuclear bound state which corresponds to I = 0 $\Lambda(1405)$ quasi-bound state lying at the energy 27MeV below the K^-p threshold. $\Lambda(1405)$ was discovered in 1961 as a broad peak in the $(\pi\Sigma)^0$ invariant mass spectrum when the 1.15GeV/c K^- beam passes through a hydrogen bubble chamber, and was attributed to a baryon species with strangeness S = -1, spin-parity $J^P = (1/2)^-$, and isospin I = 0. $\Lambda(1405)$ resonance has been interpreted as the quasi-bound state of $\bar{K}N$ embedded in $\pi\Sigma$ continuum. Particle Data Group (PDG) interpreted on the experimental finding of K^-p bound state that it is located around 1405MeV. It also predicted that the binding energy of $\Lambda(1405)$ is (~27 MeV) below the KN threshold level and has a level width of 50.0 ± 2.0 MeV.

In contrast, chiral dynamics claimed that the $\Lambda(1405)$ is described by a superposition of two resonance states . One state located around 1420 MeV couples mainly to the $\bar{K}$ N channel while the other one sitting around 1390 MeV with a 130 MeV width couples strongly to the $\pi\Sigma$ channel. In accordance with the chiral dynamics model, the binding energy ~15 MeV to 30MeV and width 50.0MeV of $\Lambda(1405)$ are obtained.

It is still a controversial issue about the position of $\bar{K}N$ quasi bound state whether $\Lambda(1405)$ or $\Lambda(1420)$. Therefore, investigating the parameter sets of $\Lambda(1405)$ is an interesting topic. In our research, the parameter set 1 for $\Lambda(1405)$ which satisfies the Particle Data Group's data and set 2 for $\Lambda(1420)$ which satisfies the Chiral Model's prediction were constructed.

Mathematical Formulation

Our purpose is to investigate the potential parameters of $\bar{K}N$ interaction which can produce the binding energy and decay width of $\Lambda(1405)$ as well as the scattering length of $\bar{K}N$ interaction. The potential parameters of $\bar{K}N$ system for single channel and the scattering length of $\bar{K}N$ interaction were calculated. Then, the optical potential for $\bar{K}N$-$\pi\Sigma$ coupled channel system was constructed by using Fesbach Theory. In our calculation, the Yukawa-type separable potential was used.

1. Calculation of Two-Body System for Single Channel

In single channel, the Schrödinger equation of two-body system in terms of separable non-local potential operator can be expressed as:

$$-\frac{\hbar^2}{2\mu}\nabla^2\psi(\vec{r})+g(\vec{r})U_0\int d\vec{r}' g(\vec{r}')\psi(\vec{r}')=\frac{\hbar^2}{2\mu}\kappa^2\psi(\vec{r}) \tag{1.1}$$

where $E=\frac{\hbar^2\kappa^2}{2\mu}$; κ = complex wave vector

$U_0=\langle 0|V|0\rangle$ = ground state potential

and $g(\vec{r})=$ the Yukawa type form factor $=\sqrt{\frac{\pi}{2}}\frac{\tilde{\Lambda}^2}{r}e^{-\tilde{\Lambda}r}$; $\tilde{\Lambda}=\sqrt{N}\Lambda$ in which N = 1 for Yukawa-type, N = 2 for expotential-type , N = ∞ for Gaussian-type potential and so on.

Here, Λ is the range parameter which depends on the mass of exchanged boson particle, i.e., $\Lambda=\frac{m_B}{\hbar c}$.

Let $F=\frac{2S\Lambda^3}{A}\int_0^\infty dr' e^{-\Lambda r'}u(r')$ and $S=2\pi^2\frac{\mu}{\hbar^2}\Lambda U_0$ is the strength parameter. Then, equation(1.1) becomes;

$$-\frac{d^2}{dr^2}u(r)+Fe^{-\Lambda r}=\kappa^2 u(r) \tag{1.2}$$

The solution with outgoing wave boundary condition is obtained by

$$u(r)=A\left(e^{-\Lambda r}-e^{i\kappa r}\right) \tag{1.3}$$

$$(\Lambda-i\kappa)^2=-S\Lambda^2 \tag{1.4}$$

This is the relation between the strength parameter and the momentum which means the energy of the $\bar{K}N$ system.

For bound/virtual state, i.e., S<0, the relation between strength parameter and corresponding energy value is $\kappa^2 = -\Lambda^2 \left\{\sqrt{-S} \pm 1\right\}^2$,

$$E = -\frac{\hbar^2}{2\mu}\Lambda^2\left\{\sqrt{-S} \mp 1\right\}^2 \quad \begin{cases} (-)\, bound\, state \\ (+)\, virtual\, state \end{cases} \tag{1.5}$$

For resonance state, i.e. S>0; $\kappa^2 = \left\{-i\Lambda \pm \Lambda\sqrt{S}\right\}^2$

$$\mathrm{E_{Res}} = \frac{\hbar^2}{2\mu}\Lambda^2(\mathrm{S}-1) \mp \mathrm{i}\frac{\hbar^2}{2\mu}\Lambda^2.2\sqrt{\mathrm{S}} \tag{1.6}$$

where, $\mathrm{E_{Res}} = \mathrm{E_R} - \mathrm{i}\frac{\Gamma}{2}$ and $\mathrm{E_{Res}} = \frac{\hbar^2\kappa^2}{2\mu}$

$$\mathrm{E_R} = \frac{\hbar^2}{2\mu}\Lambda^2(\mathrm{S}-1) \tag{1.7}$$

$$\Gamma = \pm\frac{\hbar^2}{2\mu}\Lambda^2.4\sqrt{\mathrm{S}} \quad \begin{cases} (+)\ \text{resonance state} \\ (-)\ \text{anti-resonance state} \end{cases} \tag{1.8}$$

For scattering state, the wave function solution of equation(1.2) is

$$\mathrm{u(r)} = \mathrm{A}\left(\mathrm{e}^{-\Lambda \mathrm{r}}\sin\delta - \sin(\kappa \mathrm{r} + \delta)\right) \tag{1.9}$$

where A is the amplitude and δ is the phase shift. The scattering length of the $\bar{\mathrm{K}}\mathrm{N}$ interaction can be derived as below:

$$\therefore \mathrm{a} = -\frac{2\mathrm{S}}{(1+\mathrm{S})}.\frac{1}{\Lambda} \tag{1.10}$$

2. Construction of Optical Potential by Feshbach Theory

According to the Feshbach theory, the projection operators are P for channel I and Q for channel II.

$$\mathrm{PHP.P\psi + PVQ}\frac{1}{(E-QHQ)}\mathrm{QVP.P\psi = E.P\psi} \tag{2.1}$$

This is the channel I equation for coupled-channel system.

According to Feshbach theory, introducing the optical potential as;

$$U^{opt}(E\ ) = PVP + PVQ\,\frac{1}{(E-QHQ)}\,QVP$$

Since $\langle \vec{k}' | U^{opt}(E) | \vec{k} \rangle = g(\vec{k}')\, U_0^{opt}(E)\ g(\vec{k})$ (2.2)

$$U_0^{opt}(E) = U_0^{I} + U_0^{I\,II} G_{II}(E)\ U_0^{II\,I} \tag{2.3}$$

$G_{II}(E)$ is a loop integral of channel II for coupled channel system and can be written as; $G_{II}(E) = \int d\vec{q}\, g(q) G_{II}(q,q) g(q)$

$$\therefore G_{II}(E) = -2\pi^2 \frac{\mu_{II}}{\hbar^2} \Lambda \left(\frac{\Lambda}{\Lambda - i\kappa'} \right)^2 \tag{2.4}$$

Then, the optical potential equation can be written as;

$$U_0^{opt}(E) = U_0^{I} - \left(2\pi^2 \frac{\mu_{II}}{\hbar^2} \Lambda \right) U_0^{I\,II} \left(\frac{\Lambda}{\Lambda - i\kappa'} \right)^2 U_0^{II\,I} \tag{2.5}$$

$$U_0^{opt}(E) = U_0^{I} - \left(2\pi^2 \frac{\mu_{II}}{\hbar^2} \Lambda \right) U_0^{I\,II} \frac{\Lambda^2 (\Lambda + i\kappa'^*)^2}{(\Lambda - i\kappa')^2 (\Lambda + i\kappa'^*)^2} U_0^{II\,I} \tag{2.6}$$

Since $\kappa' = \kappa'_R + i\kappa'_I$ and $\kappa'^* = \kappa'_R - i\kappa'_I$,

$$\mathrm{Re}\, U_0^{opt}(E) = U_0^{I} - \left(2\pi^2 \frac{\mu_{II}}{\hbar^2} \Lambda \right) U_0^{I\,II} \frac{\Lambda^2 \left\{ (\Lambda + i\kappa'_I)^2 - \kappa'^2_R \right\}}{\left\{ (\Lambda + i\kappa'_I)^2 + \kappa'^2_R \right\}^2} U_0^{II\,I} \tag{2.7a}$$

$$\mathrm{Im}\, U_0^{opt}(E) = -\left(2\pi^2 \frac{\mu_{II}}{\hbar^2} \Lambda \right) U_0^{I\,II} \frac{2\Lambda^2 \kappa'_R (\Lambda + i\kappa'_I)}{\left\{ (\Lambda + i\kappa'_I)^2 + \kappa'^2_R \right\}^2} U_0^{II\,I} \tag{2.7b}$$

The above two equations are the real and imaginary parts of the optical potential for channel I, i.e., $\bar{K}N$ channel.

In addition, the potential parameter of the optical potential can be written as;

$$S^{opt}(E) = S_I - S_{I\,II} \left(\frac{\Lambda}{\Lambda - i\kappa'} \right)^2 S_{II\,I} \tag{2.8}$$

$$S^{opt}(E) = S_{I} - S_{I\,II} \frac{\Lambda^2 (\Lambda + i\kappa'^*)^2}{(\Lambda - i\kappa')^2 (\Lambda + i\kappa'^*)^2} S_{II\,I}$$

We get the real and imaginary parts of the strength parameter of that equation.

$$\mathrm{Re}\; S^{opt}(E) = S_{I} - S_{I\,II} \frac{\Lambda^2 \left\{(\Lambda + i\kappa'_I)^2 - \kappa'^2_R\right\}}{\left\{(\Lambda + i\kappa'_I)^2 + \kappa'^2_R\right\}^2} S_{II\,I} \qquad (2.9a)$$

$$\mathrm{Im}\; S^{opt}(E) = -S_{I\,II} \frac{2\Lambda^2 \kappa'_R (\Lambda + i\kappa'_{\ I})}{\left\{(\Lambda + i\kappa'_I)^2 + \kappa'^2_R\right\}^2} S_{II\,I} \qquad (2.9b)$$

where, $E = \frac{\hbar^2}{2\mu_I}\kappa^2 = \frac{\hbar^2}{2\mu_{II}}\kappa'^2 - \Delta M$ and $\Delta M \approx 103\mathrm{MeV/c}^2$ is the threshold mass difference. It is note that $\kappa = -\kappa$ if $\mathrm{Im}\,\kappa < 0$ for channel I ($\because \mathrm{Im}\,\kappa > 0$ for bound state) and $\kappa' = -\kappa'$ if $\mathrm{Im}\,\kappa > 0$ for channel II ($\because \mathrm{Im}\,\kappa < 0$ for virtual state). It is also assumed that $\Lambda_1 = \Lambda_2 = \Lambda$, $S_{12} = S_{21}$ and the potential of channel II $U_0^{II} = 0$.In constructing a phenomenological potential, to determine the parameters of the potential is very important. Thus, the strength parameters and scattering length of the optical potential for channel I in coupled-channel system.

By applying the potential parameter set 1 and set 2 which satisfy the PDG data and chiral group's data, binding energy and level width of Λ(1405) were also numerically determined with the use of power inverse iteration method. Since the parameters are complex value, the real part of energy eigen value gives the binding energy of $\bar{K}N$ system and the imaginary part gives the level width of Λ(1405) resonance, respectively.

Results and Discussion

In our research, the parameter set 1 for Λ(1405) which satisfies the Particle Data Group's data and set 2 for Λ(1420) which satisfies the Chiral Model's prediction are constructed. Firstly, the potential parameters for single channel are calculated by solving the Schrödinger equation with Yukawa-type separable potential. The scattering length of $\bar{K}N$ interaction is also calculated.

According to the results of single channel calculation, the strength parameter set 1 are S = -1.527 - i0.233 and U = -3.063 -i0.466 while the strength parameter set 2 are S = -1.384 – i0.238 and U = -2.476 – i0.428.

To compute the potential parameters for the coupled-channel system, assuming that the exchanged particles of channel I and channel II are the same, i.e., $\Lambda_1 = \Lambda_2 = \Lambda$, and the interaction between $\pi\Sigma$ - $\pi\Sigma$: $V_{22} = 0$ and varying the mass of boson exchanged particles, potential parameters and scattering length are obtained which are summarized in table 1 and table 2. We have selected the potential parameter set 1 and set 2 whose scattering lengths are consistent with the experimental scattering length [5]. According to table 1 and 2, it is found that our calculated scattering length is consistent with experimental value at the boson energy mass m_B = 600 MeV/c^2 for parameter set 1 and at m_B = 670 MeV/c^2 for parameter set 2. Our calculated results are summarized in table 3.

With the single channel potential parameter set 1 and set 2, binding energies and level widths of $\bar{K}N$ system are determined by solving the Schrödinger equation numerically with Gaussian basis treatment. To obtain the energy eigen value, power inverse iteration method is used. Our calculated results are compared with other theoretical and experimental results, which are illustrated in table 4. It is found that our calculated scattering length and energy are in good agreement with experimental and other theoretical results.

Table 1: The parameters sets 1 for Λ(1405) varying with the masses of exchanged bosons (BE = 27MeV/c^2, level width = 25.0MeV/c^2, $S_{12} = S_{21}$ and S_{22} = 0.0)

M_B (MeV/c^2)	Λ (fm^{-1})	S^{opt}	S_{11}	S_{12}	U_0 for channel I (MeV.fm^3)	$a^{I=0}_{\bar{K}N}$ (fm)
140.0	0.710	-3.943 -i1.629	-4.224	1.676	-33.900 -i14.010	-3.552 +i0.406
280.0	1.420	-2.249 -i0.608	-1.852	0.892	-9.665 -i2.615	-2.322 +i0.444
400.0	2.029	-1.827 -i0.383	-1.381	0.772	-5.498 -i1.151	-1.969 +i0.454
500.0	2.534	-1.644 -i0.290	-1.191	0.730	-3.958 -i0.698	-1.808 +i0.458
600.0	**3.041**	**-1.527 -i0.233**	**-1.074**	**0.707**	**-3.063 -i0.466**	**-1.702 +i0.461**
610.0	3.091	-1.518 -i0.228	-1.065	0.705	-2.995 -i0.450	-1.693 -i0.461
620.0	3.142	-1.509 -i0.224	-1.056	0.703	-2.929 -i0.434	-1.685 -i0.461
630.0	3.193	-1.500 -i0.220	-1.048	0.702	-2.865 -i0.419	-1.677 -i0.461
640.0	3.243	-1.492 -i0.215	-1.039	0.700	-2.805 -i0.405	-1.669 -i0.461
650.0	3.294	-1.483 -i0.212	-1.031	0.699	-2.746 -i0.392	-1.661 -i0.461
700.0	3.547	-1.446 -i0.194	-0.996	0.692	-2.486 -i0.333	-1.627 +i0.462
770.0	3.902	-1.403 -i0.174	-0.954	0.685	-2.192 -i0.271	-1.586 +i0.463
800.0	4.057	-1.386 -i0.166	-0.939	0.682	-2.085 -i0.250	-1.571 +i0.463
900.0	4.564	-1.341 -i0.145	-0.896	0.675	-1.793 -i0.194	-1.528 +i0.464

Experimental value of scattering length [5], $a^{I=0}$ = - (1.70 ± 0.07) + i(0.68 ±0.04)

Table 2: The parameters set 2 varying with the masses of exchanged bosons (BE = 15.0MeV/c^2, level width = 25.0 MeV/c^2, S_{12} = S_{21}, S_{22} = 0.0)

M_B (MeV/c^2)	Λ (fm^{-1})	S^{opt}	S_{11}	S_{12}	U_0 for channel I (MeV.fm^3)	$a^{I=0}_{\bar{K}N}$ (fm)
140.0	0.710	-3.202 -i1.792	-3.615	1.853	-27.526 -i15.402	-3.589 +i0.627
280.0	1.420	-1.977 -i0.689	-1.579	0.954	-8.499 -i2.964	-2.372 +i0.679
400.0	2.029	-1.658 -i0.439	-1.188	0.817	-4.989 -i1.322	-2.024 i0.692
500.0	2.534	-1.517 -i0.335	-1.033	0.770	-3.651 -i0.807	-1.864 +i0.697
600.0	3.041	-1.425 -i0.270	-0.937	0.745	-2.859 -i0.542	-1.759 +i0.700
650.0	3.294	-1.391 -i0.247	-0.908	0.736	-2.575 -i0.456	-1.719 +i0.701
660.0	3.345	-1.384 -i0.242	-0.896	0.735	-2.524 -i0.442	-1.712 +i0.702
670.0	3.395	-1.384 -i0.238	-0.890	0.733	-2.476 -i0.428	-1.704 +i0.702
680.0	3.446	-1.372 -i0.234	-0.884	0.732	-2.429 -i0.414	-1.698 +i0.702
690.0	3.497	-1.367 -i0.230	-0.878	0.730	-2.384 -i0.401	-1.691 +i0.702
700.0	3.547	-1.361 -i0.226	-0.873	0.729	-2.340 -i0.389	-1.685 +i0.702
770.0	3.902	-1.327 -i0.203	-0.839	0.721	-2.074 -i0.317	-1.644 +i0.703
800.0	4.057	-1.314 -i0.194	-0.827	0.718	-1.976 -i0.293	-1.629 +i0.703
900.0	4.564	-1.277 -i0.170	-0.792	0.711	-1.708 -i0.228	-1.586 +i0.704

Experimental value of scattering length [5], $a^{I=0}$ = - (1.70 ± 0.07) + i(0.68 ±0.04)

Table 3 : Our calculated results

	m_B (MeV/c^2)	Λ (fm^{-1})	S^{opt}	S_{11}	S_{12}	U_0 for channel I (MeV.fm^3)
Parameter set 1	600.0	3.041	-1.527 -i0.233	-1.074	0.707	-3.063 -i0.466
Parameter set 2	670.0	3.395	-1.384 -i0.238	-0.890	0.733	-2.476 -i0.428

Table 4: Comparison of our calculated results and other theoretical and experimental results

	Λ(fm^{-1})	$a^{I=0}$ (fm)	BE- i $\frac{\Gamma}{2}$ (MeV)
Y. Akaishi and T. Yamazaki for Λ(1405) [6]	3.90	- 1.76 + i0.46	-27.0 - i20
Thida Oo for Λ(1405) [7]	3.91	- 1.67 + i0.42	-26.84 - i19.93
PDG for Λ(1405) [3]	-	-	~ - 27.0 – i25.0 ± 1.0
Chiral prediction for Λ(1420) [4]	-	-	~15 - i25.0
Experimental value of scattering length [5]	-	- (1.70 ± 0.07) + i(0.68 ± 0.04)	-
Our calculated parameter set 1 for Λ(1405)	3.04	- 1.702 + i0.461	-26.76 – i24.84
Our calculated parameter set 2 for Λ(1420)	3.40	- 1.698 + i0.702	-14.77 - i24.80

Acknowledgements

Author gratefully acknowledges to Dr Khin Swe Myint, Rector (Rtd.), Emeritus Professor, Department of Physics, University of Mandalay, for her excellent advice, kindness and valuable discussion throughout this research.

Author also thanks to Professor Dr Lei Lei Win, Head of Department of Physics, Professor Dr Kalyar Thwe and Professor Dr Yee Yee Oo, Department of Physics, University of Mandalay, for their permission, and valuable discussion.

Author deeply expresses her gratitude to her supervisor, Dr Thida Wint, lecturer, Department of Physics, University of Mandalay, for her patience, kindness, excellent advice and continuous guidance throughout this research work.

References

A.D. Martin, Nucl. Phys. B **179** (1981)33

K.A. Olive et al., “Particle Data Group”, Chin. Phys. C 38(2014)090001.

M. Iwasaki et al., Phys. Rev. Lett. **78** (1997)3067;

M. H. Alston et al., Phys. Rev. Lett **6** (1961) 698.

Thida Oo, “PhD Thesis”, University of Mandalay (2010)

T. Hyodo, “Proceeding of Sendai International Symposium” (2008).

T. M. Ito et al., Phys. Rev. C **58** (1998) 2366.

Y. Akaishi, T. Yamazaki, Phys. Rev. C **65** (2002) 044005.